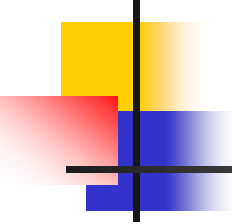


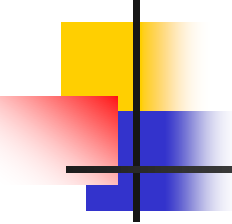
CHAPTER 5:

HYPOTHESIS TESTS ABOUT THE MEAN AND PROPORTION



HYPOTHESIS TESTS: AN INTRODUCTION

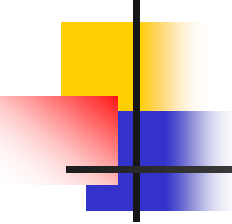
- Two Hypotheses
- Rejection and Nonrejection Regions
- Two Types of Errors
- Tails of a Test



Two Hypotheses

Definition

A *null hypothesis* is a claim (or statement) about a population parameter that is assumed to be true until it is declared false.



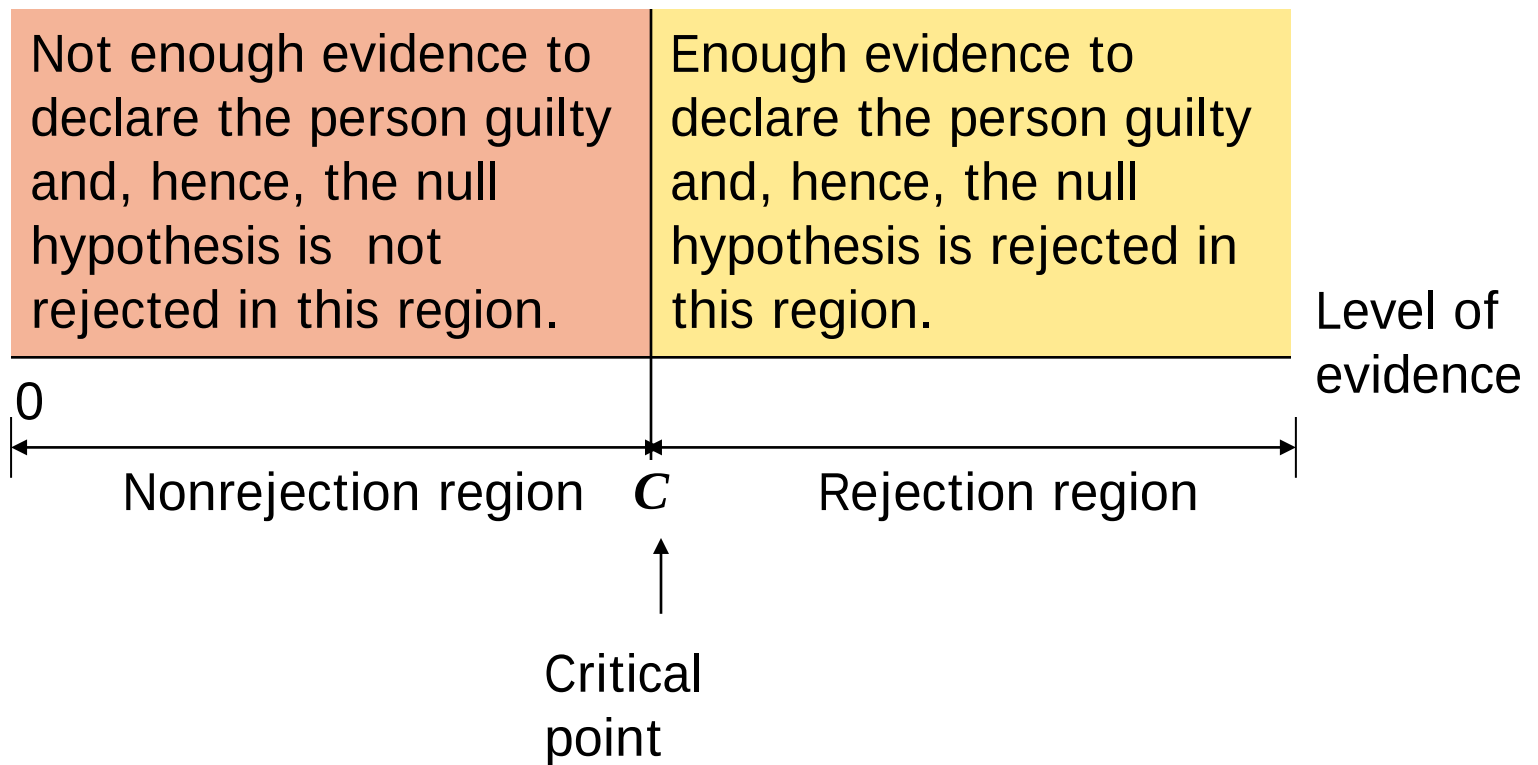
Two Hypotheses cont.

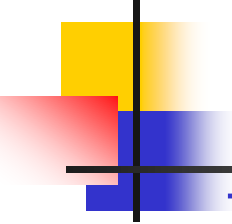
Definition

An *alternative hypothesis* is a claim about a population parameter that will be true if the null hypothesis is false.

Rejection and Nonrejection Regions

Figure 9.1 Nonrejection and rejection regions for the court case.

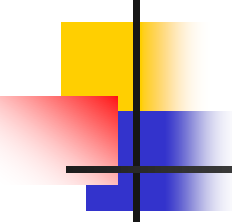




Two Types of Errors

Table 9.1

		Actual Situation	
		The Person Is Not Guilty	The Person Is Guilty
Court's decision	The person is not guilty	Correct decision	Type II or β error
	The person is guilty	Type I or α error	Correct decision



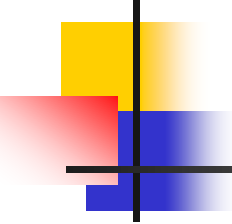
Two Types of Errors cont.

Definition

A *Type I error* occurs when a true null hypothesis is rejected. The value of α represents the probability of committing this type of error; that is,

$$\alpha = P(H_0 \text{ is rejected} \mid H_0 \text{ is true})$$

The value of α represents the *significance level* of the test.



Two Types of Errors cont.

Definition

A *Type II error* occurs when a false null hypotheses is not rejected. The value of β represents the probability of committing a Type II error; that is

$$\beta = P(H_0 \text{ is not rejected} \mid H_0 \text{ is false})$$

The value of $1 - \beta$ is called the *power of the test*. It represents the probability of not making a Type II error.

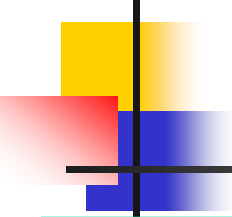
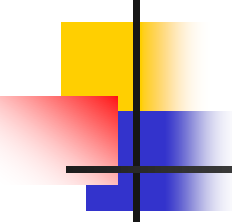


Table 9.2

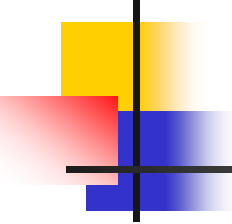
		Actual Situation	
		H_0 Is True	H_0 Is False
Decision	Do not reject H_0	Correct decision	Type II or β error
	Reject H_0	Type I or α error	Correct decision



Tails of a Test

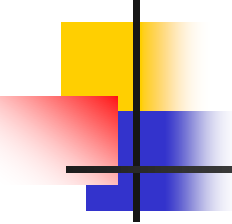
Definition

A *two-tailed test* has rejection regions in both tails, a *left-tailed test* has the rejection region in the left tail, and a *right-tailed test* has the rejection region in the right tail of the distribution curve.



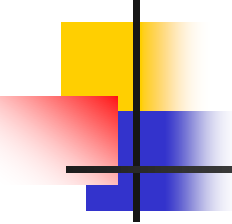
A Two-Tailed Test

- According to the U.S. Bureau of the Census, the mean family size in the United States was 3.18 in 1998. A researcher wants to check whether or not this mean has *changed* since 1998.
- The mean family size has changed if it has either increased or decreased during the period since 1998. This is an example of a two tailed test.



A Two-Tailed Test cont.

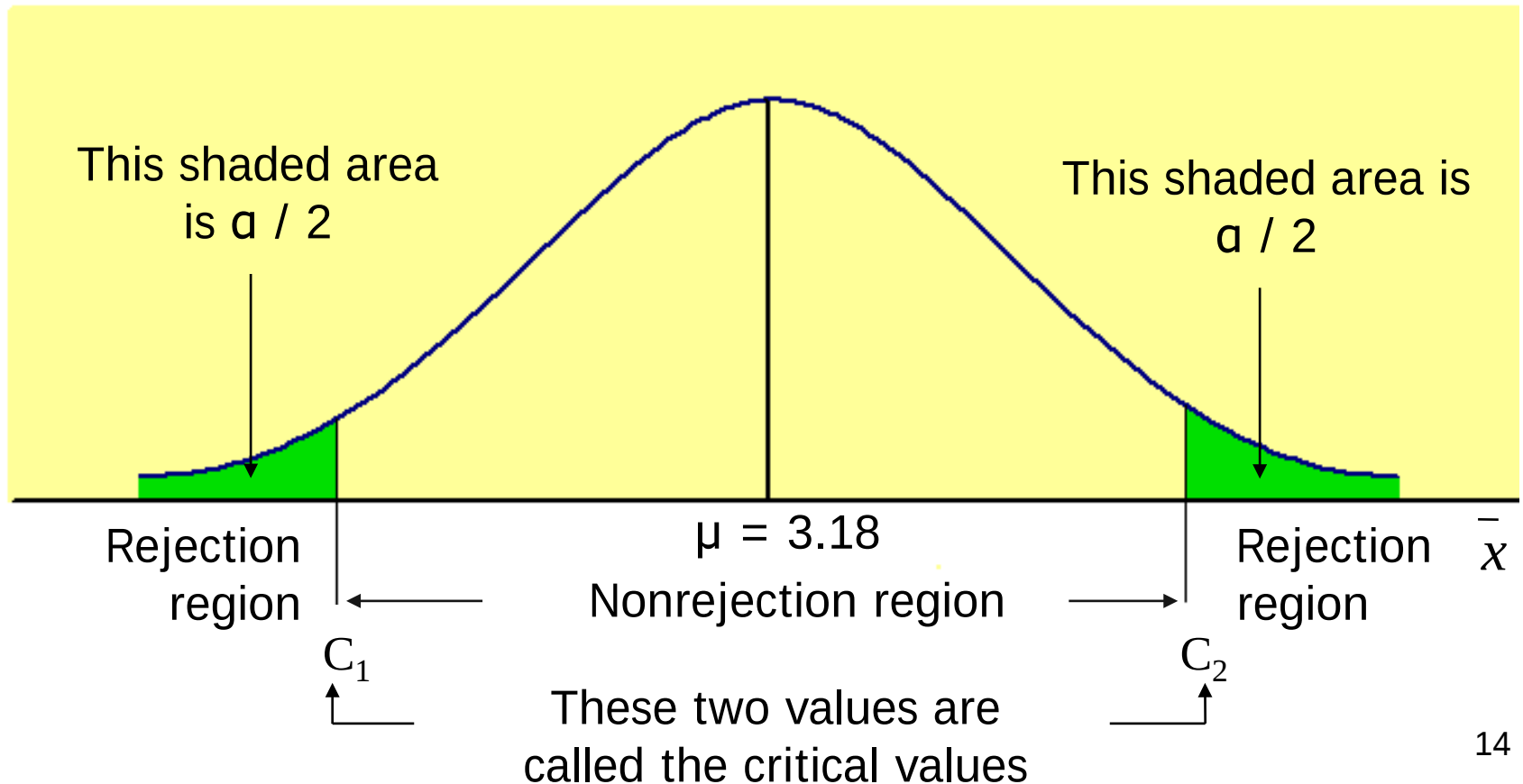
- Let μ be the current mean family size for all families. The two possible decisions are
 - $H_0 : \mu = 3.18$ (The mean family size has not changed)
 - $H_1 : \mu \neq 3.18$ (The mean family size has changed)

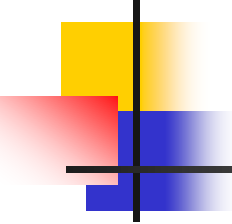


A Two-Tailed Test cont.

- Whether a test is two – tailed or one – tailed is determined by the sign in the alternative hypothesis.
- If the alternative hypothesis has a *not equal* to ($\neq$) sign, it is a two – tailed test.

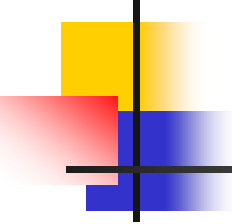
Figure 9.2 A two-tailed test.





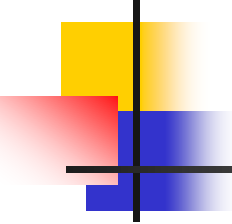
A Left-Tailed Test

A soft-drink company claims that the cans, on average, contain 12 ounces of soda. However, if these cans contain less than the claimed amount of soda, then the company can be accused of cheating. Suppose a consumer agency wants to test whether the mean amount of soda per can is *less than* 12 ounces.



A Left-Tailed Test cont.

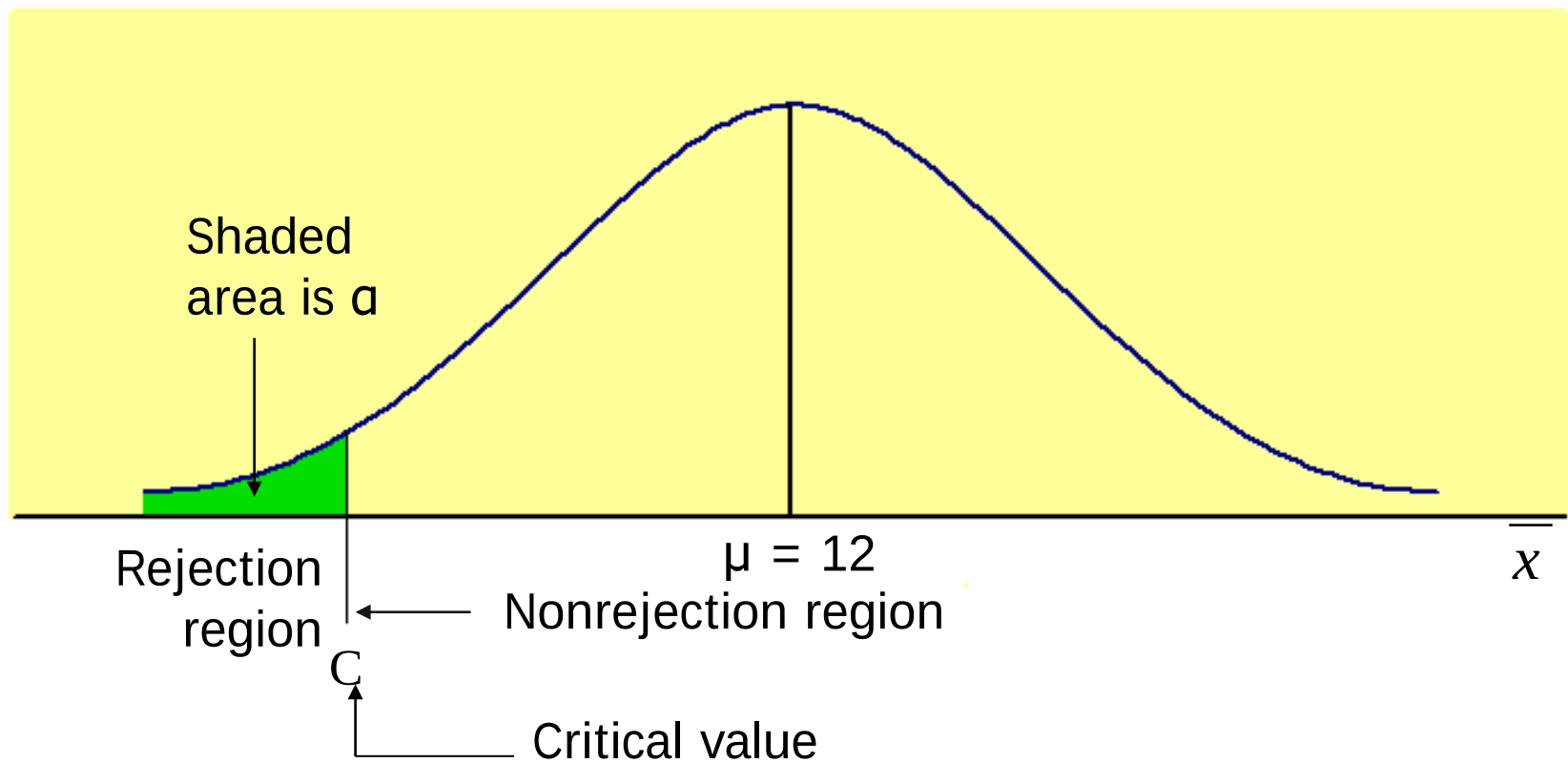
- Let μ be the mean amount of soda in all cans. The two possible decisions are
 - $H_0 : \mu = 12$ ounces (The mean is not less than 12 ounces)
 - $H_1 : \mu < 12$ ounces (The mean is less than 12 ounces)

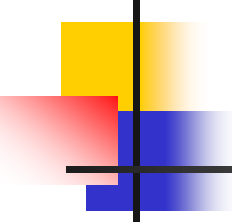


A Left-Tailed Test cont.

When the alternative hypothesis has a less than ($<$) sign, the test is always left – tailed.

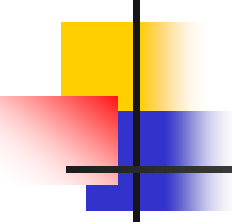
Figure 9.3 A left-tailed test.





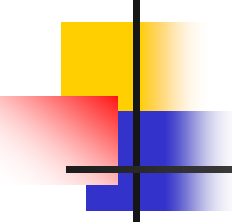
A Right-Tailed Test

According to a 1999 study by the American Federation of Teachers, the mean starting salary of school teachers in the U.S. was \$25,735 during 1997 – 98. Suppose we want to test whether the current mean starting salary of all school teachers in the United States is *higher than* \$25,735.



A Right-Tailed Test cont.

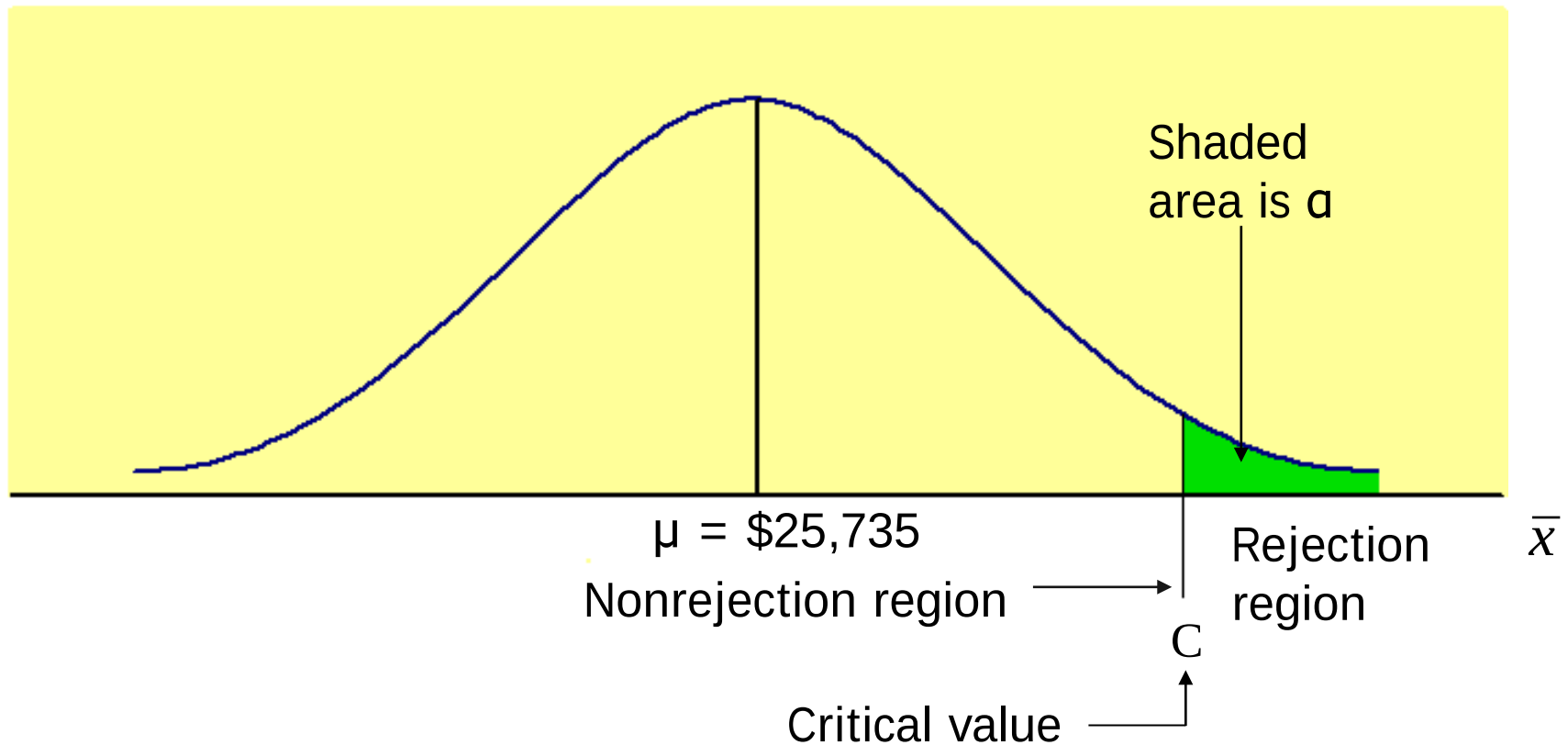
- Let μ be the current mean starting salary of school teachers in the United States. The two possible decisions are
 - $H_0 : \mu = \$25,735$ (The current mean starting salary is not higher than \$25,735)
 - $H_1 : \mu > \$25,735$ (The current mean starting salary is higher than \$25,735)



A Right-Tailed Test cont.

When the alternative hypothesis has a greater than ($>$) sign, the test is always right – tailed.

Figure 9.4 A right-tailed test.



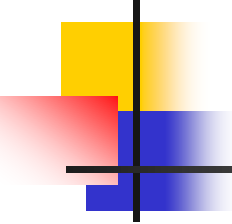
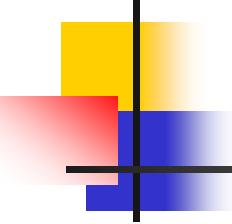


Table 9.3

	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in the null hypothesis H_0	=	= or $\geq$	= or $\leq$
Sign in the alternative hypothesis H_1	$\neq$	<	>
Rejection region	In both tails	In the left tail	In the right tail



HYPOTHESIS TESTS ABOUT μ FOR LARGE SAMPLES USING THE p - VALUE APPROACH

Definition

The p – value is the smallest significance level at which the null hypothesis is rejected.

Figure 9.5

The p - value for a right-tailed test.

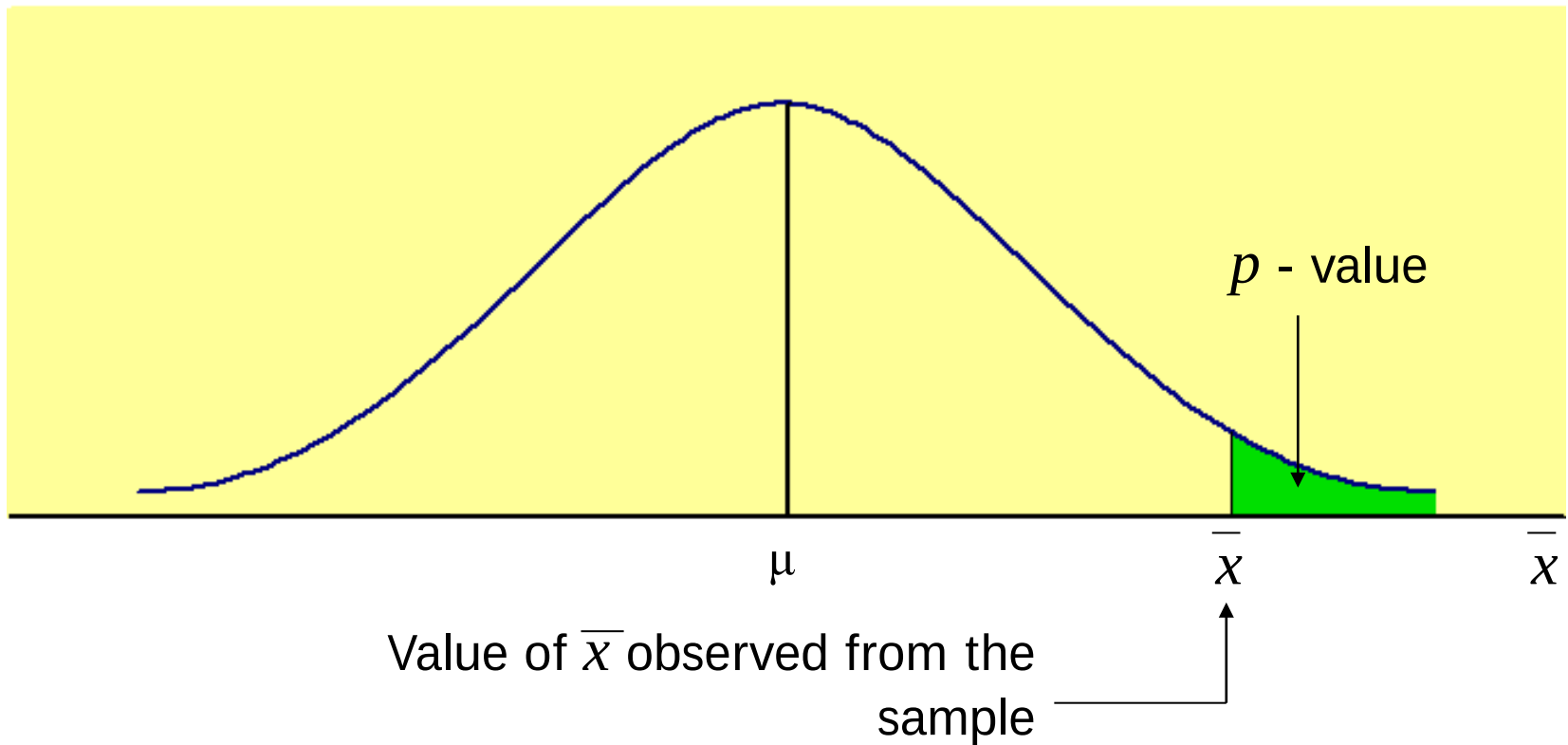
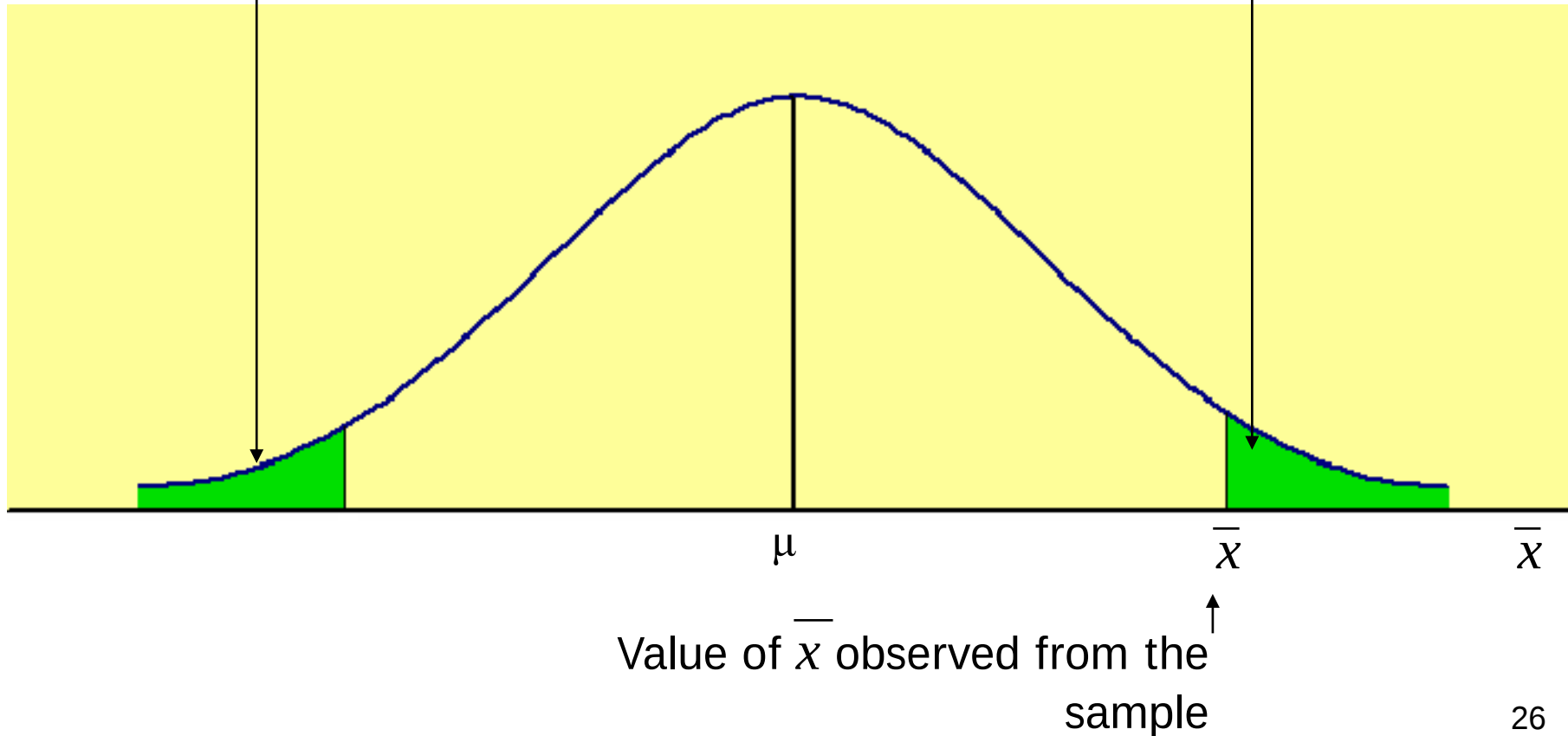
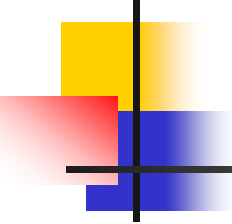


Figure 9.6 The p - value for a two-tailed test.

The sum of these two areas
gives the p - Value





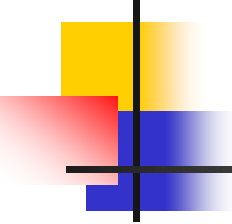
Calculating the z Value for $\bar{x}$

For a large sample, the value of z for $\bar{x}$ for a test of hypothesis about μ is computed as follows:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{if } \sigma \text{ is known}$$

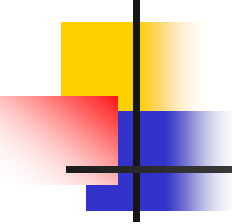
$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{if } \sigma \text{ is not known}$$

where $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ and $s_{\bar{x}} = s / \sqrt{n}$



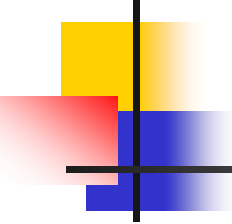
Calculating the z Value for $\bar{x}$ cont.

The value of z calculated for $\bar{x}$ using the formula is also called the *observed value of z* .



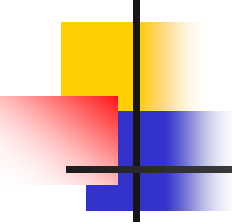
Steps to Perform a Test of Hypothesis Using the p – Value Approach

1. State the null and alternative hypotheses.
2. Select the distribution to use.
3. Calculate the p – value.
4. Make a decision.



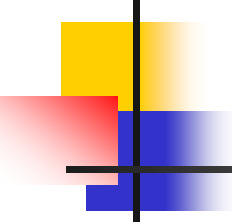
Example 9-1

The management of Priority Health Club claims that its members lose an average of 10 pounds or more within the first month after joining the club. A consumer agency that wanted to check this claim took a random sample of 36 members of this health club and found that they lost an average of 9.2 pounds within the first month of membership with a standard deviation of 2.4 pounds. Find the p – value for this test. What will your decision be if $\alpha = .01$? What if $\alpha = .05$?



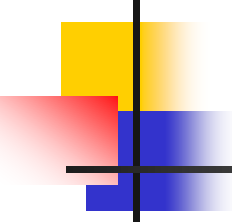
Solution 9-1

- $H_0: \mu \geq 10$ (The mean weight lost is 10 pounds or more)
- $H_1: \mu < 10$ (The mean weight lost is less than 10)



Solution 9-1

- The sample size is large ($n > 30$)
- Therefore, we use the normal distribution to make the test and to calculate the p – value.



Solution 9-1

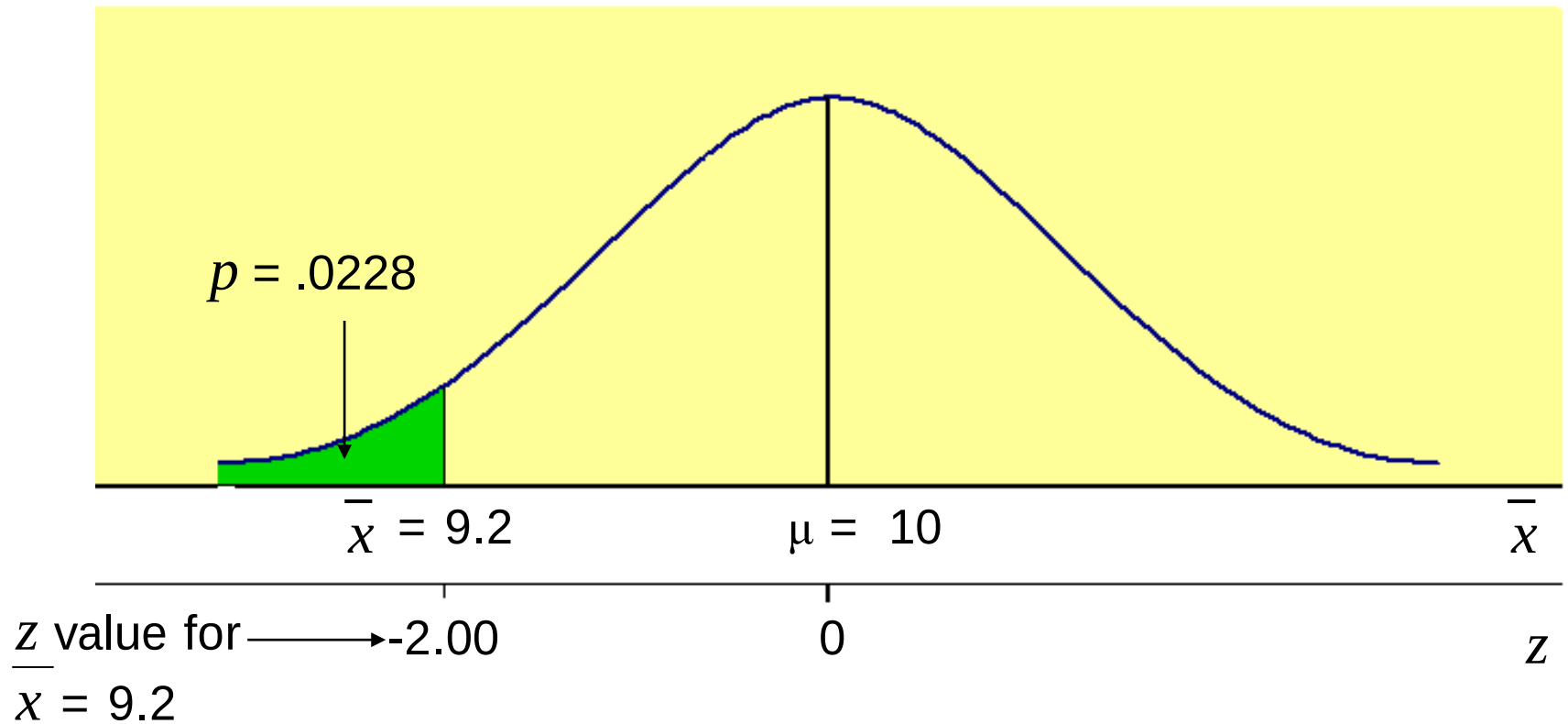
$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.4}{\sqrt{36}} = .40$$

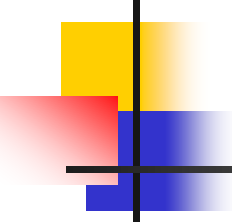
$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{9.2 - 10}{.40} = -2.00$$

$$p - \text{value} = .0228$$

Figure 9.7

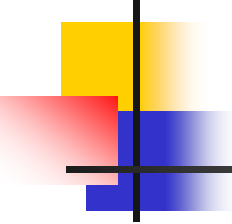
The p – value for a left-tailed test.





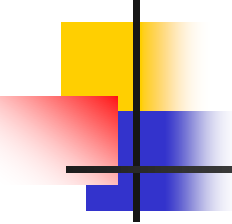
Solution 9-1

- The p – value is .0228
- $\alpha = .01$
 - It is less than the p – value
- Therefore, we do not reject the null hypothesis
- $\alpha = .05$
 - It is greater than the p – value
- Therefore, we reject the null hypothesis.



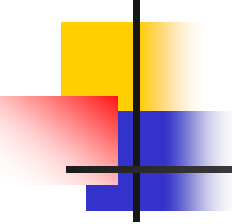
Example 9-2

At Canon Food Corporation, it took an average of 50 minutes for new workers to learn a food processing job. Recently the company installed a new food processing machine. The supervisor at the company wants to find if the mean time taken by new workers to learn the food processing procedure on this new machine is different from 50 minutes.



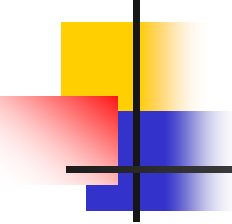
Example 9-2

A sample of 40 workers showed that it took, on average, 47 minutes for them to learn the food processing procedure on the new machine with a standard deviation of 7 minutes. Find the p – value for the test that the mean learning time for the food processing procedure on the new machine is different from 50 minutes. What will your conclusion be if $\alpha = .01$.



Solution 9-2

- $H_0: \mu = 50$ minutes
- $H_1: \mu \neq 50$ minutes



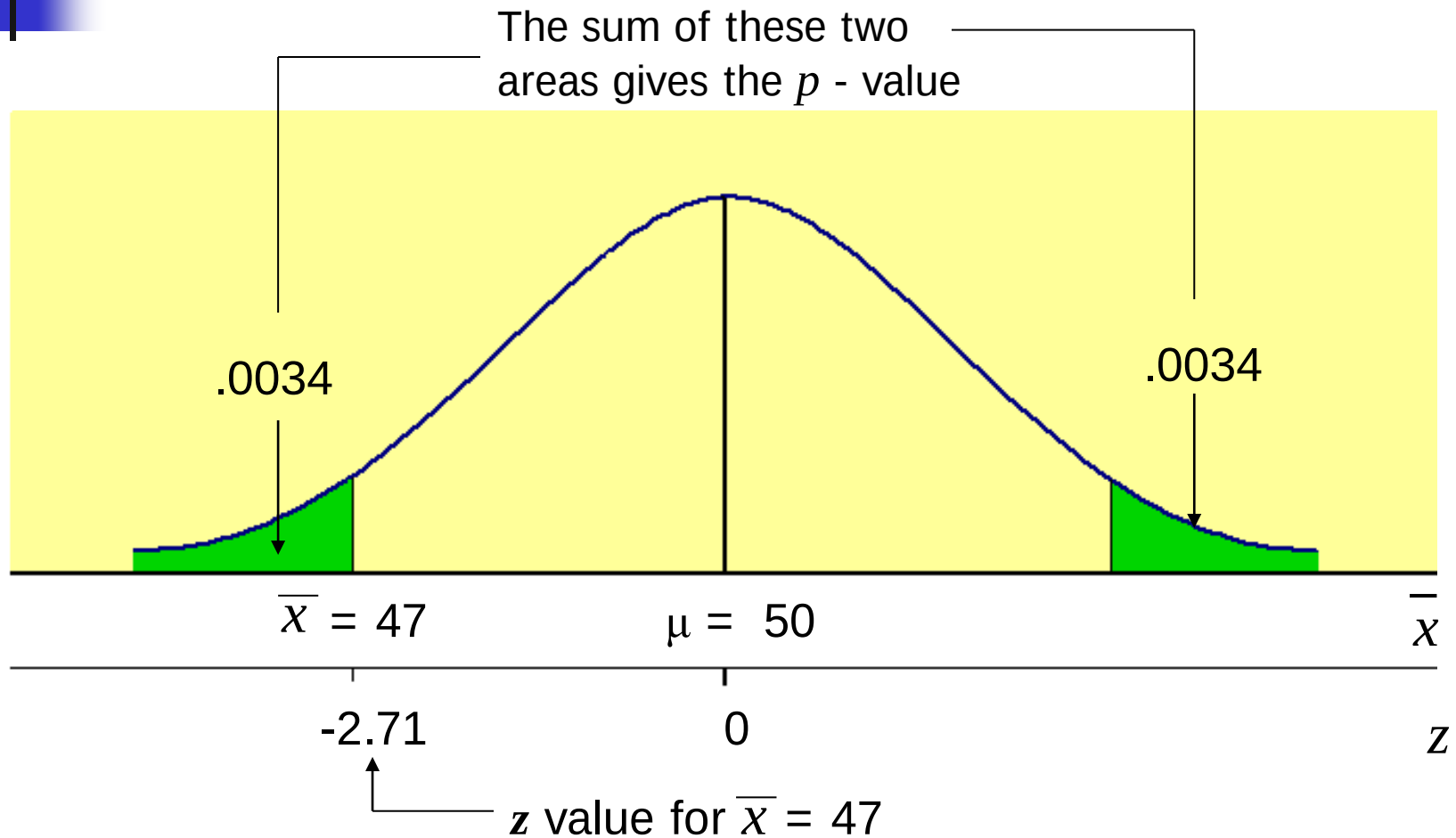
Solution 9-2

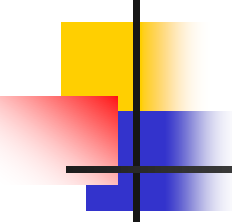
$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{7}{\sqrt{40}} = 1.10679718 \text{ minutes}$$

$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{47 - 50}{1.10679718} = -2.71$$

- Hence, the area to the left of $z = -2.71$ is
 $.5 - .4966 = .0034$.
- Consequently, the p – value is
 $2(.0034) = \mathbf{.0068}$

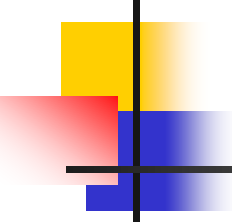
Figure 9.8 The p - value for a two-tailed test.





Solution 9-2

Because $\alpha = .01$ is greater than the p – value of .0068, we reject the null hypothesis.



HYPOTHESIS TESTS ABOUT A POPULATION MEAN: LARGE SAMPLES

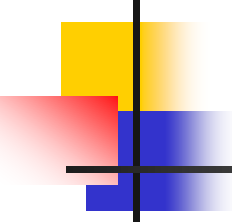
Test Statistic

In tests of hypotheses about μ for large samples, the random variable

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \quad \text{or} \quad Z = \frac{\bar{X} - \mu}{s_{\bar{X}}}$$

where $\sigma_{\bar{X}} = \sigma / \sqrt{n}$ and $s_{\bar{X}} = s / \sqrt{n}$

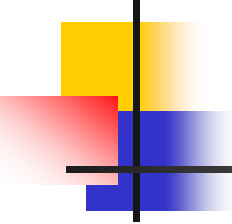
is called the **test statistic**. The test statistic can be defined as a rule or criterion that is used to make the decision whether or not to reject the null hypothesis.



HYPOTHESIS TESTS ABOUT A POPULATION MEAN: LARGE SAMPLES cont.

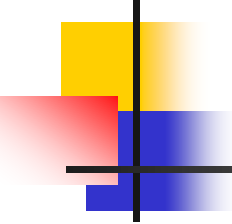
Steps to Perform a Test of Hypothesis with Predetermined α

1. State the null and alternative hypotheses.
2. Select the distribution to use.
3. Determine the rejection and nonrejection regions.
4. Calculate the value of the test statistic.
5. Make a decision.



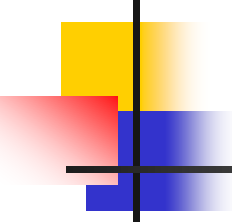
Example 9-3

The TIV Telephone Company provides long-distance telephone service in an area. According to the company's records, the average length of all long-distance calls placed through this company in 1999 was 12.44 minutes. The company's management wanted to check if the mean length of the current long-distance calls is different from 12.44 minutes.



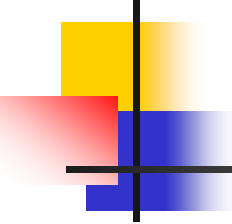
Example 9-3

A sample of 150 such calls placed through this company produced a mean length of 13.71 minutes with a standard deviation of 2.65 minutes. Using the 5% significance level, can you conclude that the mean length of all current long-distance calls is different from 12.44 minutes?



Solution 9-3

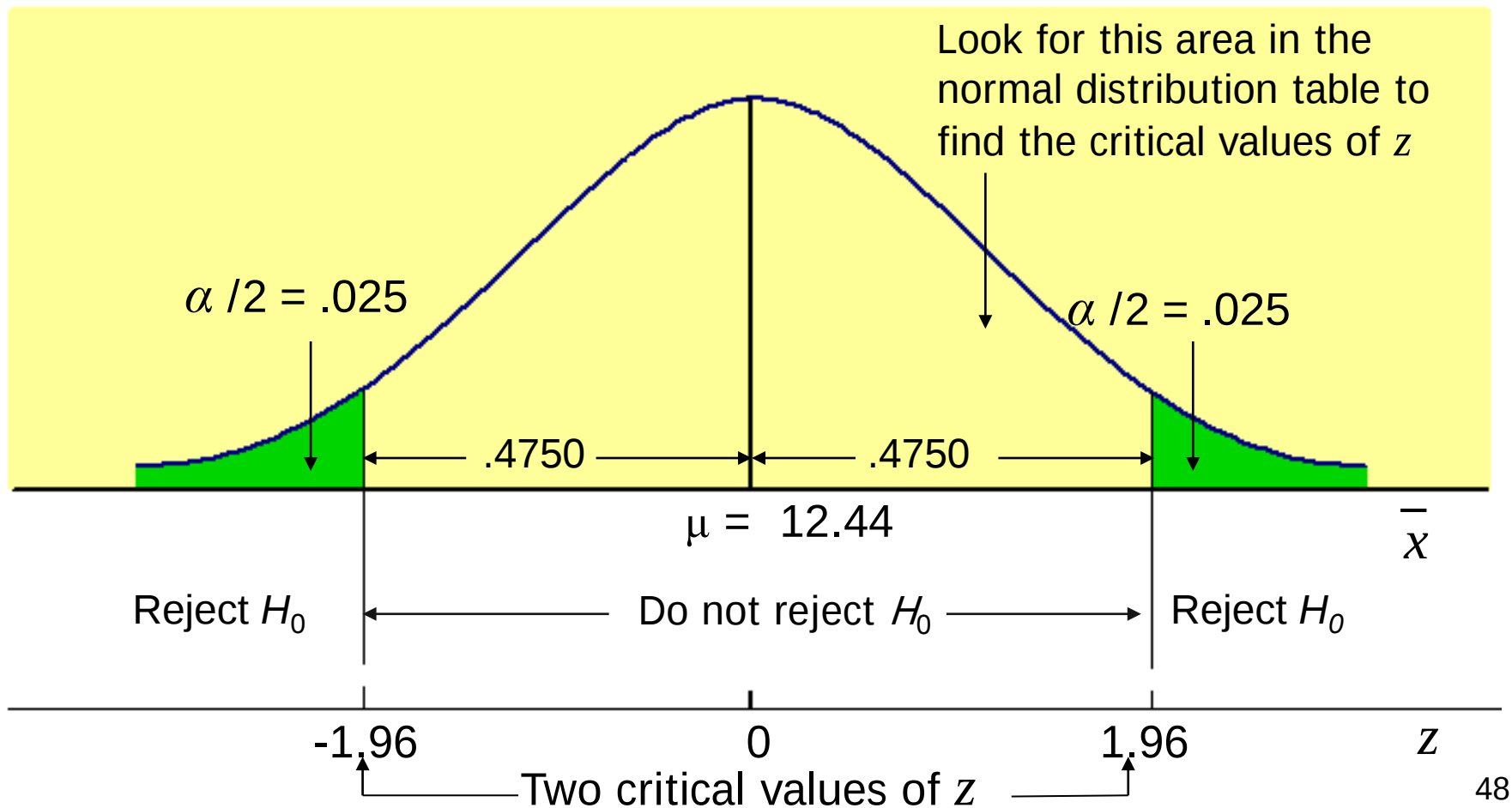
- $H_0 : \mu = 12.44$
 - The mean length of all current long-distance calls is 12.44 minutes
- $H_1 : \mu \neq 12.44$
 - The mean length of all current long-distance calls is different from 12.44 minutes

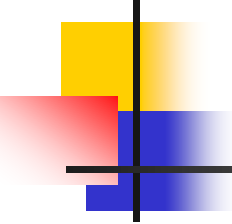


Solution 9-3

- $\alpha = .05$
- The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed
- Area in each tail = $\alpha / 2 = .05 / 2 = .025$
- The z values for the two critical points are -1.96 and 1.96

Figure 9.9





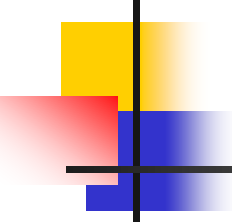
Calculating the Value of the Test Statistic

For a large sample, *the value of the test statistic z* for $\bar{x}$ for a test of hypothesis about μ is computed as follows:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \quad \text{if } \sigma \text{ is known}$$
$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{if } \sigma \text{ is not known}$$

where $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ and $s_{\bar{x}} = s / \sqrt{n}$

This value of z for $\bar{x}$ is also called the **observed value of z** .

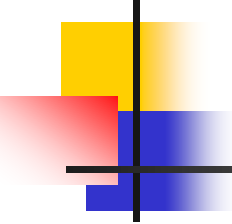


Solution 9-3

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.65}{\sqrt{150}} = .21637159$$

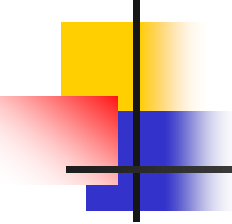
From H_0

$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{13.71 - 12.44}{.21637159} = 5.87$$



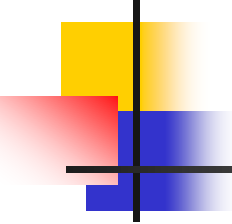
Solution 9-3

- The value of $z = 5.87$
 - It is greater than the critical value
 - It falls in the rejection region
- Hence, we reject H_0



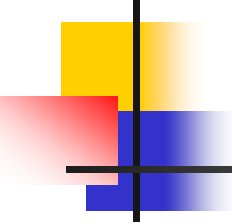
Example 9-4

According to a salary survey by National Association of Colleges and Employers, the average salary offered to computer science majors who graduated in May 2002 was \$50,352 (*Journal of Accountancy*, September 2002). Suppose this result is true for all computer science majors who graduated in May 2002.



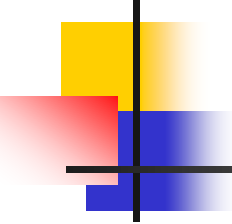
Example 9-4

A random sample of 200 computer science majors who graduated this year showed that they were offered a mean salary of \$51,750 with a standard deviation of \$5240. Using the 1% significance level, can you conclude that the mean salary of this year's computer science graduates is higher than \$50,352?



Solution 9-4

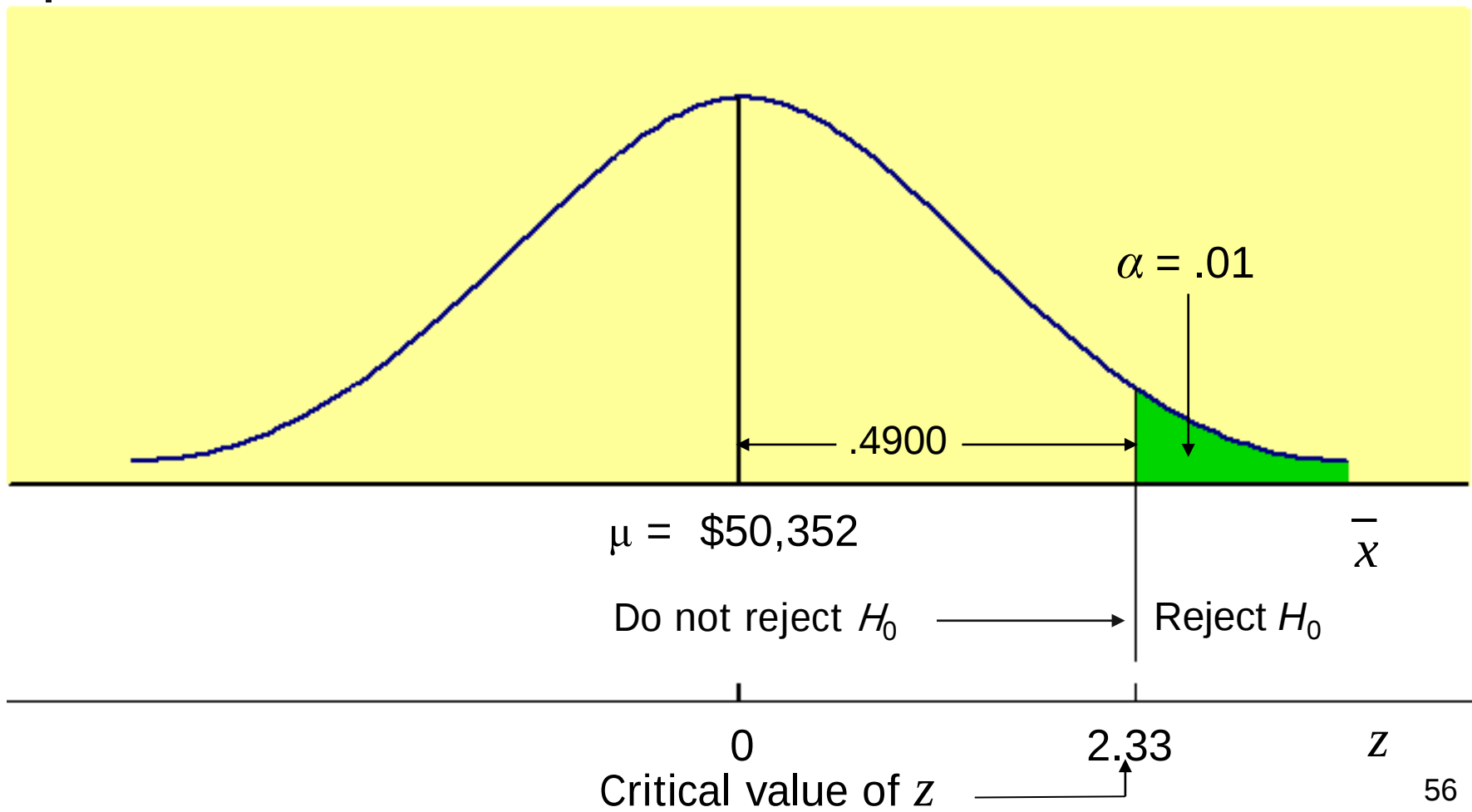
- $H_0: \mu = \$50,352$
 - The mean salary offered to this year's computer science graduates is \$50,352
- $H_1: \mu > \$50,352$
 - The mean salary offered to this year's computer science graduates is higher than \$50,352

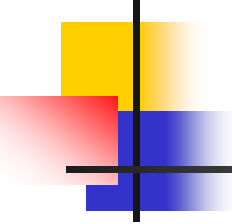


Solution 9-4

- $\alpha = .01$
- The $>$ sign in the alternative hypothesis indicates that the test is right-tailed
- Area in the right tail = $\alpha = .01$
- The critical value of z is approximately 2.33

Figure 9.10



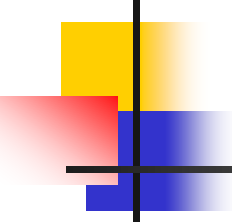


Solution 9-4

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{5240}{\sqrt{200}} = \$370.5239533$$

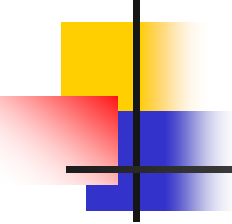
From H_0

$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{51,750 - 50,352}{370.5239533} = 3.77$$



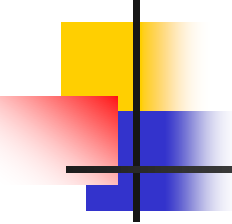
Solution 9-4

- The value of the test statistic $z = 3.77$
 - It is larger than the critical value of $z = 2.33$
 - it falls in the rejection region
- Consequently, we reject H_0



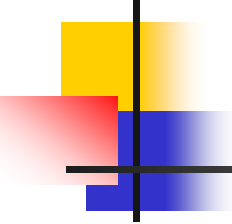
Example 9-5

The mayor of a large city claims that the average net worth of families living in this city is at least \$300,000. A random sample of 100 families selected from this city produced a mean net worth of \$288,000 with a standard deviation of \$80,000. Using the 2.5% significance level, can you conclude that the mayor's claim is false?



Solution 9-5

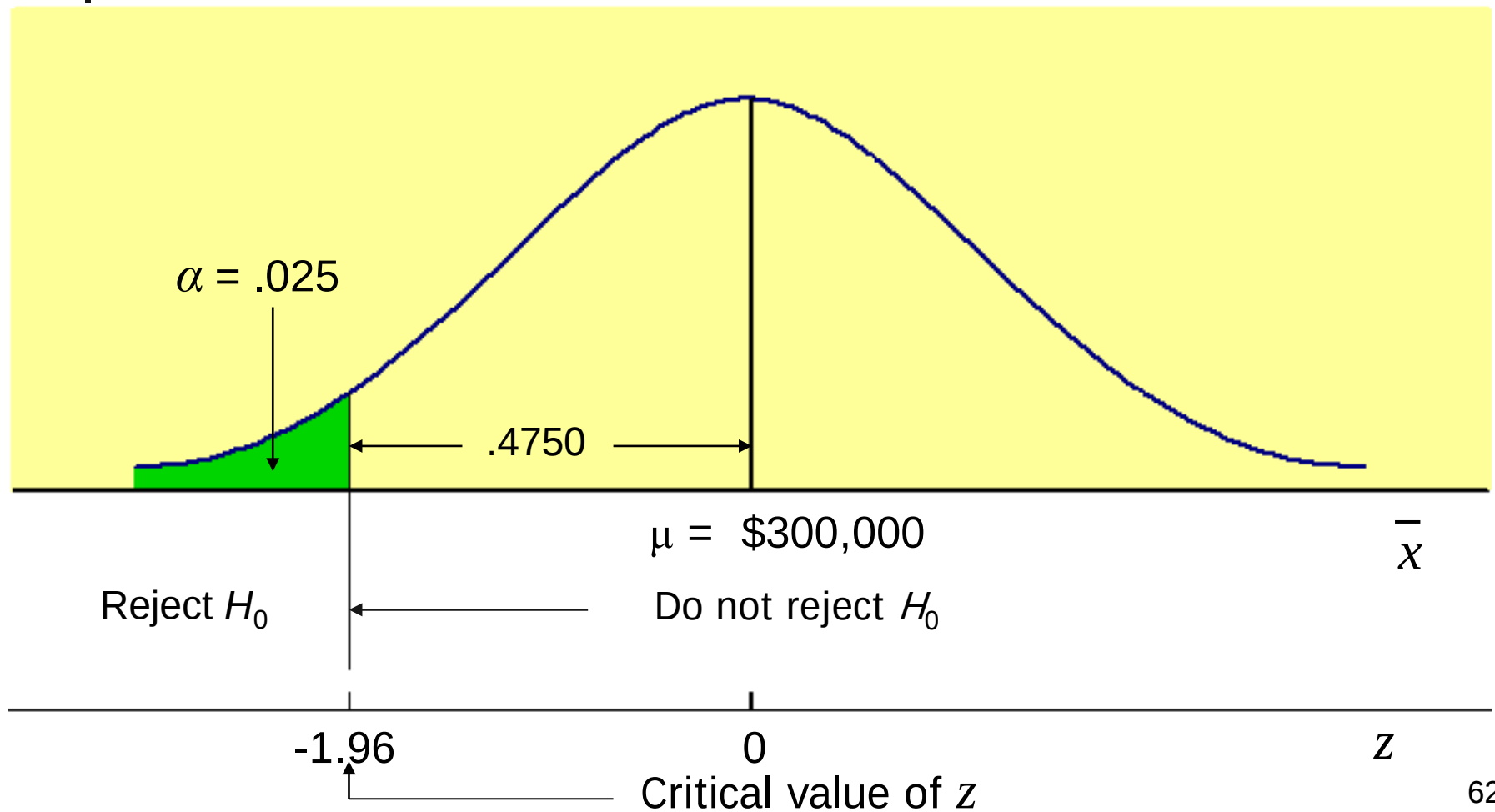
- $H_0: \mu \geq \$300,000$
 - The mayor's claim is true. The mean net worth is at least \$300,000
- $H_1: \mu < \$300,000$
 - The mayor's claim is false. The mean net worth is less than \$300,000

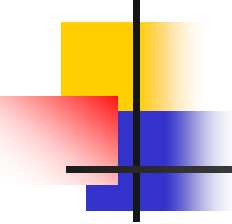


Solution 9-5

- $\alpha = .025$
- The $<$ sign in the alternative hypothesis indicates that the test is left-tailed
- The critical value of z is -1.96

Figure 9.11



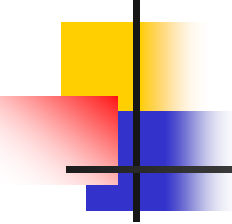


Solution 9-5

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{80,000}{\sqrt{100}} = \$8000$$

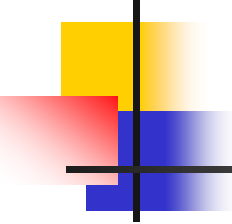
From H_0

$$z = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{288,000 - 300,000}{8000} = -1.50$$



Solution 9-5

- The value of the test statistic $z = -1.50$
 - It is greater than the critical value
 - It falls in the nonrejection region
- As a result, we fail to reject H_0



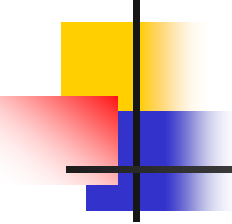
HYPOTHESIS TEST ABOUT A POPULATION MEAN: SMALL SAMPLES

Conditions Under Which the t Distribution Is
Used to Make Tests of Hypothesis About μ

The t distribution is used to conduct a *test of hypothesis about μ* if

1. The sample size is small ($n < 30$)
2. The population from which the sample is drawn is (approximately) normally distributed.
3. The population standard deviation σ is unknown.

HYPOTHESIS TEST ABOUT A POPULATION MEAN: SMALL SAMPLES cont.

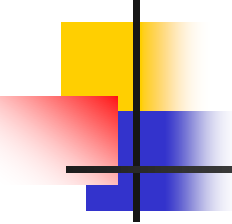


Test Statistic

The value of the *test statistic t* for the sample mean $\bar{x}$ is computed as

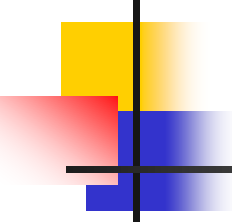
$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{where} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

The value of t calculated for $\bar{x}$ by using this formula is also called the **observed value of t**.



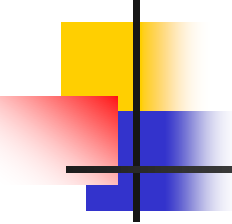
Example 9-6

A psychologist claims that the mean age at which children start walking is 12.5 months. Carol wanted to check if this claim is true. She took a random sample of 18 children and found that the mean age at which these children started walking was 12.9 months with a standard deviation of .80 month. Using the 1% significance level, can you conclude that the mean age at which all children start walking is different from 12.5 months? Assume that the ages at which all children start walking have an approximately normal distribution.



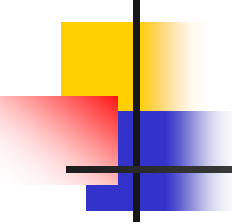
Solution 9-6

- $H_0: \mu = 12.5$
(The mean walking age is 12.5 months)
- $H_1: \mu \neq 12.5$
(The mean walking age is different from 12.5 months)



Solution 9-6

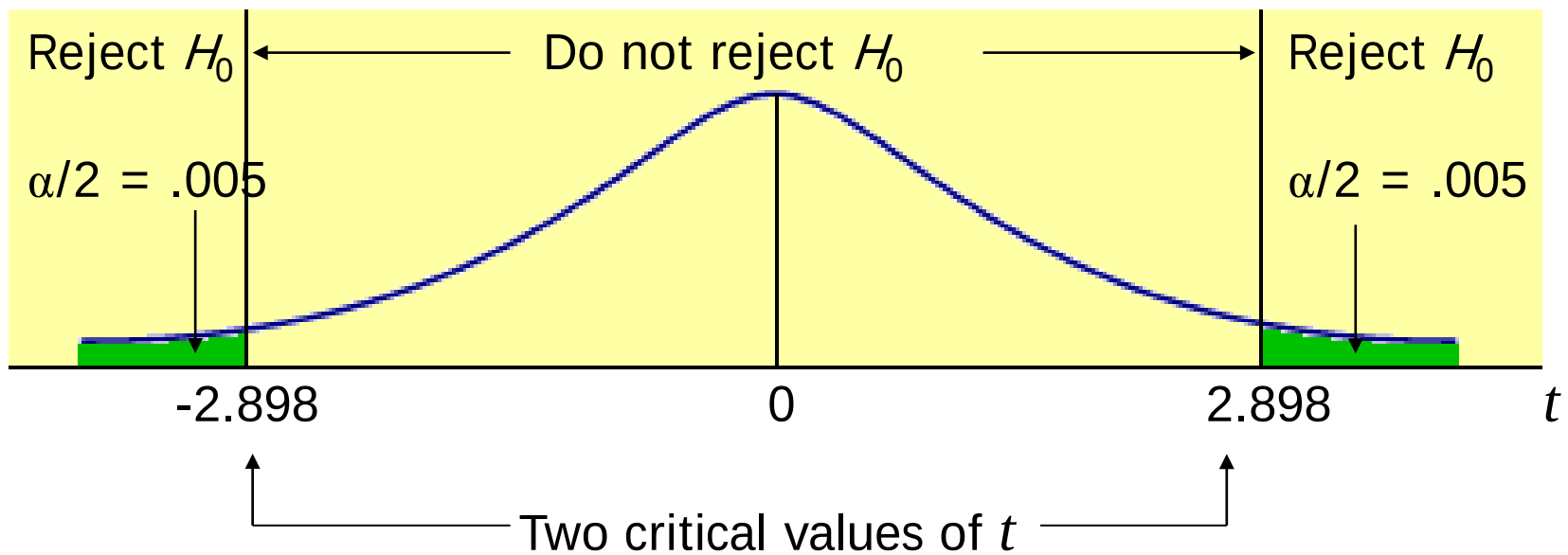
- The sample size is small
- The population is approximately normally distributed
- The population standard deviation is not known
- Hence, we use the t distribution to make the test

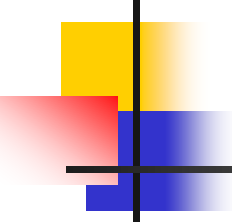


Solution 9-6

- $\alpha = .01$.
- The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed.
- Area in each tail = $\alpha / 2 = .01 / 2 = .005$
- $df = n - 1 = 18 - 1 = 17$
- Critical values of t are -2.898 and 2.898

Figure 9.12



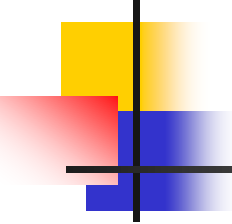


Solution 9-6

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{.8}{\sqrt{18}} = .18856181$$

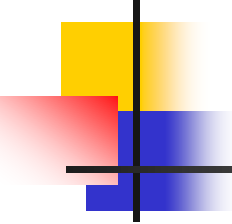
From H_0

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{12.9 - 12.5}{.18856181} = 2.121$$



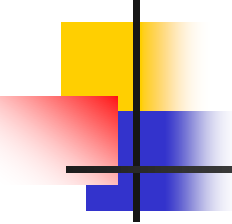
Solution 9-6

- The value of the test statistic $t = 2.121$
 - It falls between the two critical points
 - It is in the nonrejection region.
- Consequently, we fail to reject H_0 .



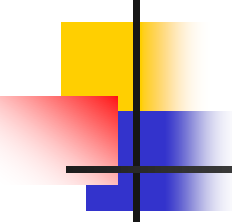
Example 9-7

Grand Auto Corporation produces auto batteries. The company claims that its top-of-the-line Never Die batteries are good, on average, for at least 65 months. A consumer protection agency tested 15 such batteries to check this claim. It found the mean life of these 15 batteries to be 63 months with a standard deviation of 2 months. At the 5% significance level, can you conclude that the claim of this company is true? Assume that the life of such a battery has an approximately normal distribution.



Solution 9-7

- $H_0: \mu \geq 65$
 - The mean life is at least 65 months
- $H_1: \mu < 65$
 - The mean life is less than 65 months



Solution 9-7

- $\alpha = .05$.
- The $<$ sign in the alternative hypothesis indicates that the test is left-tailed.
- Area in the left tail = $\alpha = .05$
- $df = n - 1 = 15 - 1 = 14$
- The critical value of t is -1.761.

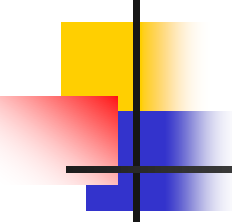
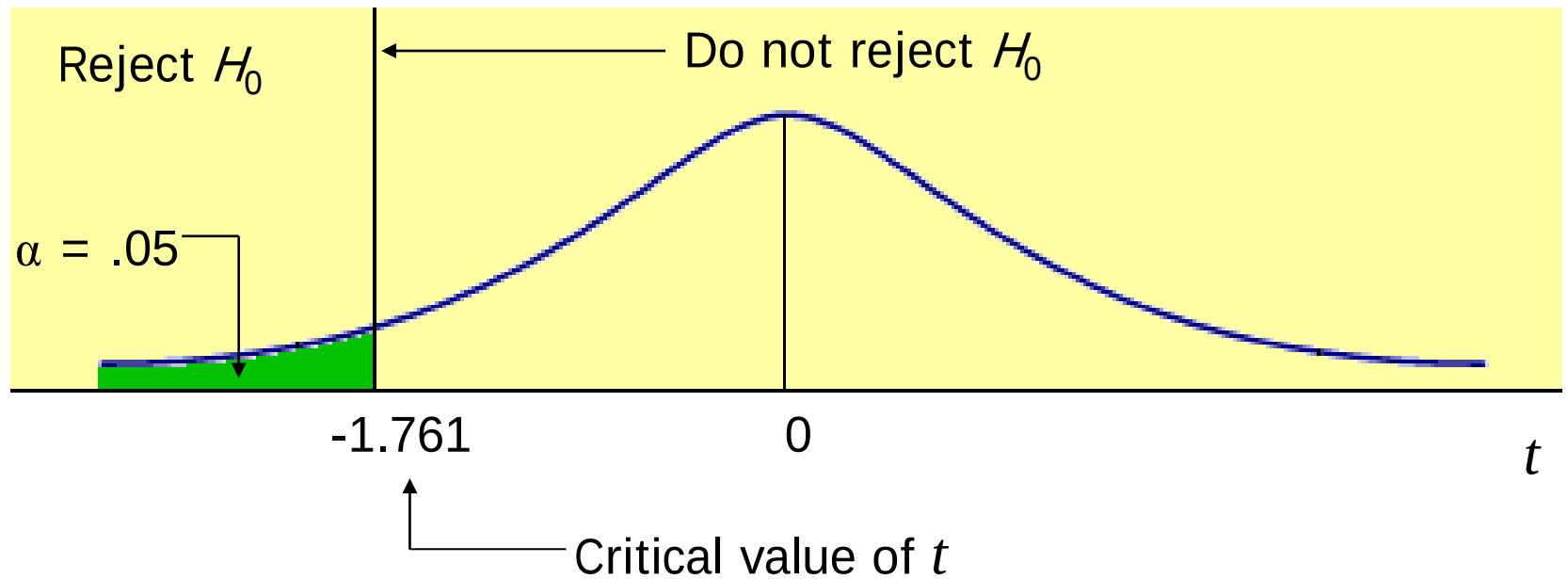
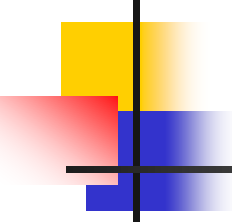


Figure 9.13



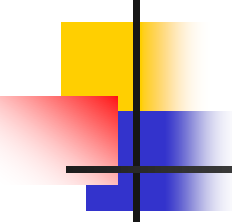


Solution 9-7

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{15}} = .51639778$$

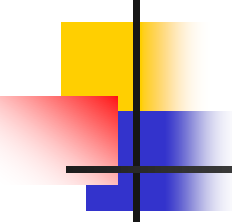
From H_0

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{63 - 65}{.51639778} = -3.873$$



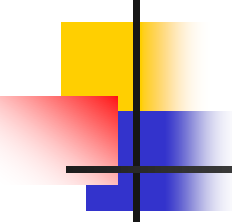
Solution 9-7

- The value of the test statistic $t = -3.873$
 - It is less than the critical value of t
 - It falls in the rejection region
- Therefore, we reject H_0



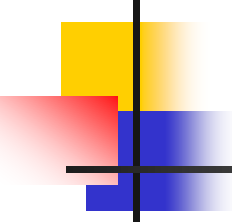
Example 9-8

The management at Massachusetts Savings Bank is always concerned about the quality of service provided to its customers. With the old computer system, a teller at this bank could serve, on average, 22 customers per hour. The management noticed that with this service rate, the waiting time for customers was too long. Recently the management of the bank installed a new computer system in the bank, expecting that it would increase the service rate and consequently make the customers happier by reducing the waiting time.



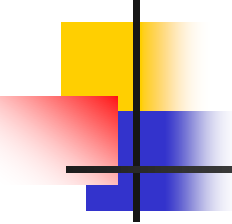
Example 9-8

To check if the new computer system is more efficient than the old system, the management of the bank took a random sample of 18 hours and found that during these hours the mean number of customers served by tellers was 28 per hour with a standard deviation of 2.5. Testing at the 1% significance level, would you conclude that the new computer system is more efficient than the old computer system? Assume that the number of customers served per hour by a teller on this computer system has an approximately normal distribution.



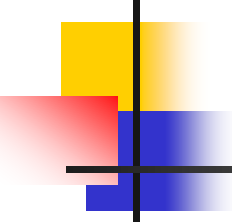
Solution 9-8

- $H_0: \mu = 22$
 - The new computer system is not more efficient
- $H_1: \mu > 22$
 - The new computer system is more efficient



Solution 9-8

- The sample size is small
- The population is approximately normally distributed
- The population standard deviation is not known
- Hence, we use the t distribution to make the test



Solution 9-8

- $\alpha = .01$
- The $>$ sign in the alternative hypothesis indicates that the test is right-tailed
- Area in the right tail = $\alpha = .01$
- $df = n - 1 = 18 - 1 = 17$
- The critical value of t is 2.567

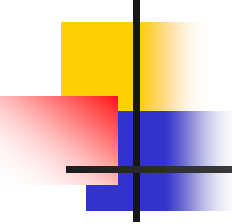
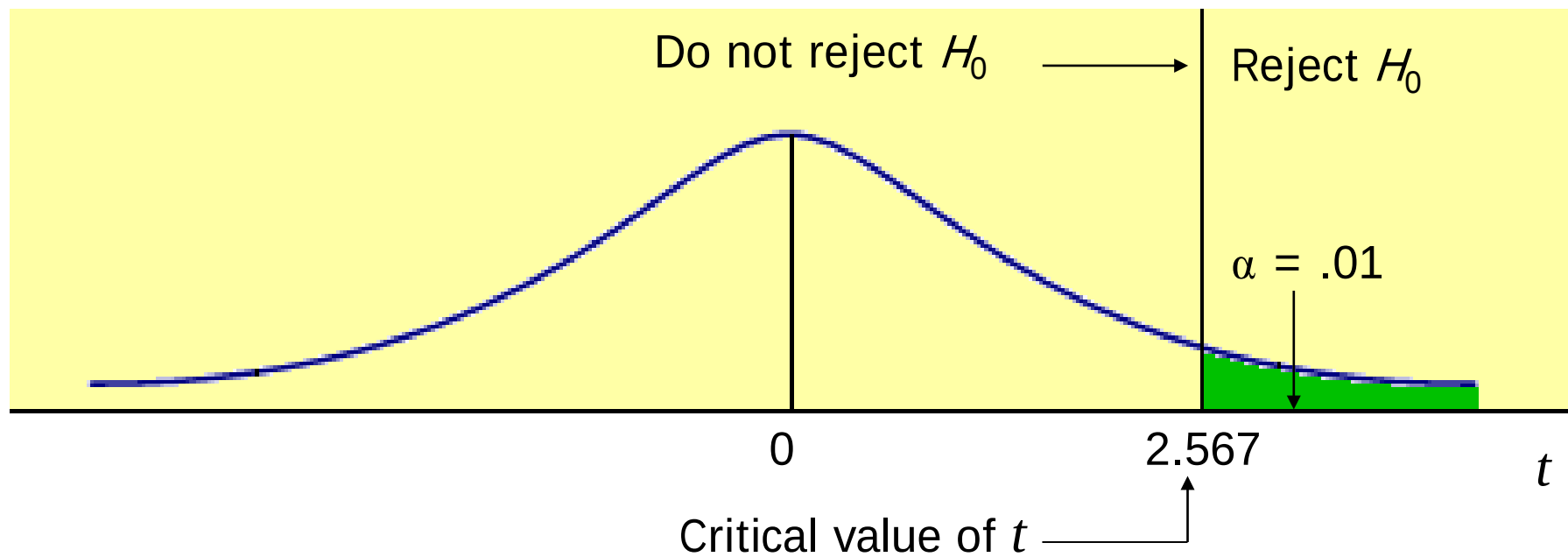
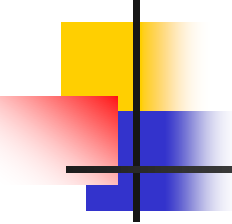


Figure 9.14



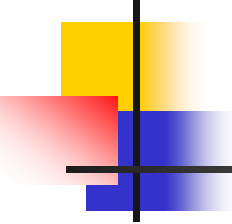


Solution 9-8

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2.5}{\sqrt{18}} = .58925565$$

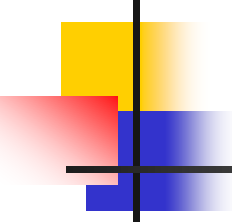
From H_0

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{28 - 22}{.58925565} = 10.182$$



Solution 9-8

- The value of the test statistic $t = 10.182$
 - It is greater than the critical value of t
 - It falls in the rejection region
- Consequently, we reject H_0

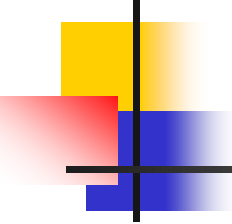


HYPOTHESIS TESTS ABOUT A POPULATION PROPORTION: LARGE SAMPLES

Test Statistic

The value of the *test statistic z* for the sample proportion, $\hat{p}$, is computed as

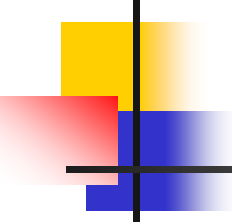
$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \quad \text{where} \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$



Test Statistic cont.

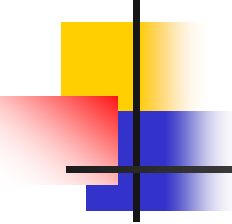
The value of p used in this formula is the one used in the null hypothesis. The value of q is equal to $1 - p$.

The value of z calculated for $\hat{p}$ using the above formula is also called the **observed value of z** .



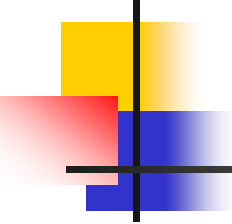
Example 9-9

In a poll by the National Center for Women and Aging at Brandeis University, 51% of the women over 50 said that aging is not as bad as they had expected (*USA TODAY*, November 19, 2002). Assume that this result holds true for the 2002 population of all women aged 50 and over. In a recent random sample of 400 women aged 50 and over, 54% said that aging is not as bad as they had expected.



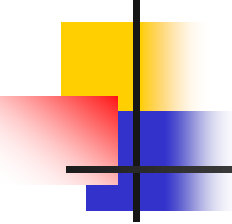
Example 9-9

Using the 1% significance level, can you conclude that the current percentage of women aged 50 and over who think that aging is not as bad as they had expected is different from that for 2002?



Solution 9-9

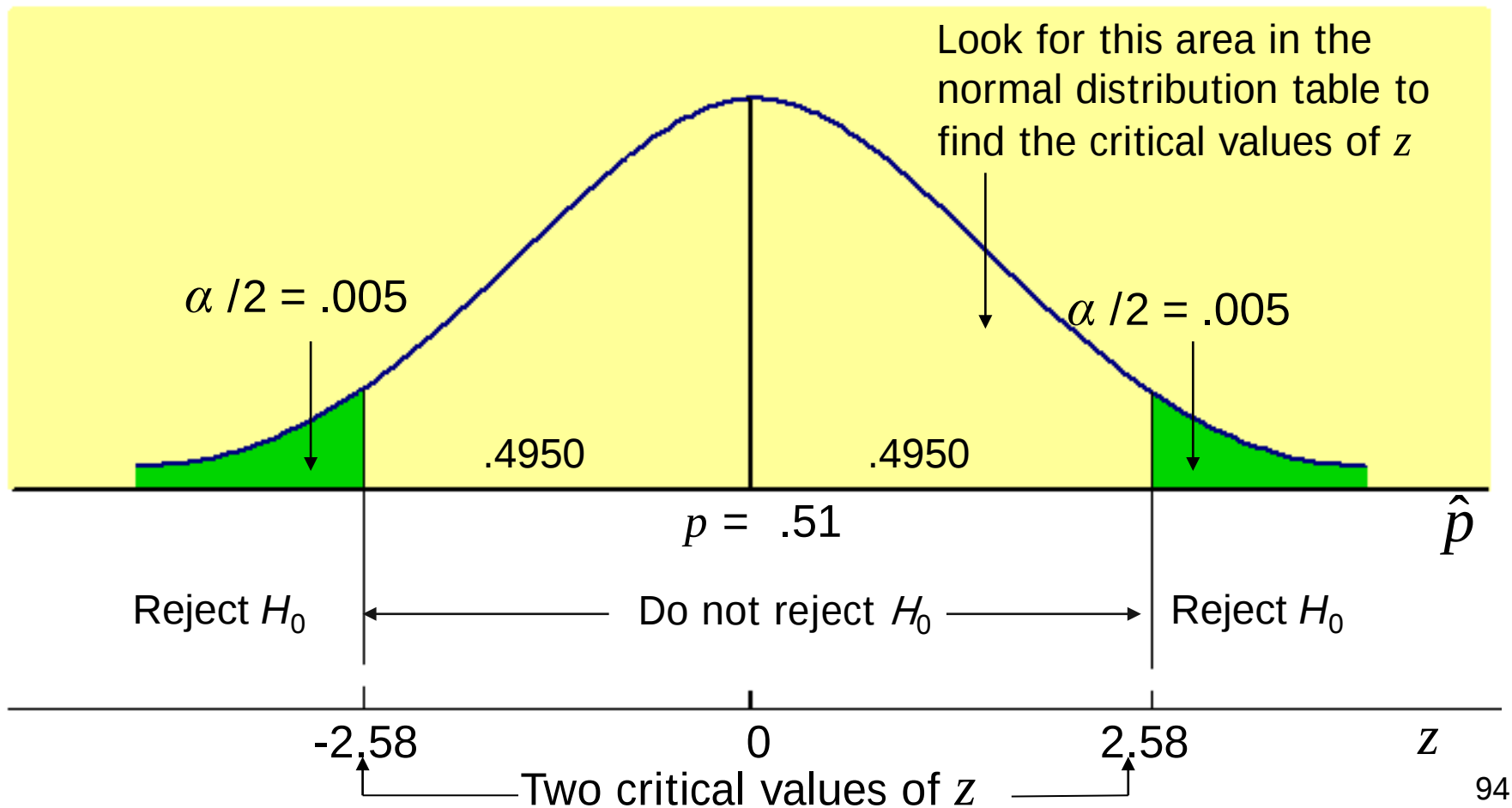
- $H_0: p = .51$
 - The current percentage is not different from that of 2002
- $H_1: p \neq .51$
 - The current percentage is different from that of 2002

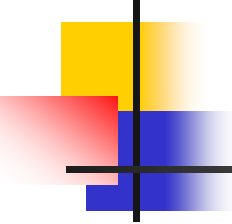


Solution 9-9

- $n = 400$, and $\hat{p} = .54$
- $\alpha = .01$
- $np = 400(.51) = 204$
- $nq = 400(.49) = 196$
- Both np and nq are greater than 5
- The sample size is large
- Consequently, we use the normal distribution to make a test about p
- The critical values of z are -2.58 and 2.58

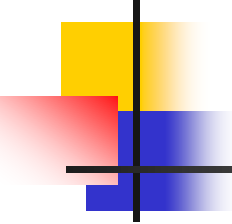
Figure 9.15





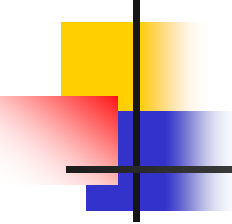
Solution 9-9

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.51)(.49)}{400}} = .02499500 \quad \text{From } H_0$$
$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.54 - .51}{.02499500} = 1.20$$



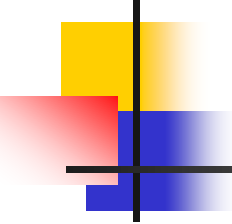
Solution 9-9

- The value of the test statistic $z = 1.20$ for $\hat{p}$ lies in the nonrejection region
- Consequently, we fail to reject H_0



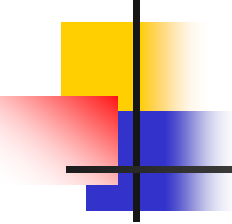
Example 9-10

When working properly, a machine that is used to make chips for calculators does not produce more than 4% defective chips. Whenever the machine produces more than 4% defective chips, it needs an adjustment. To check if the machine is working properly, the quality control department at the company often takes samples of chips and inspects them to determine if they are good or defective.



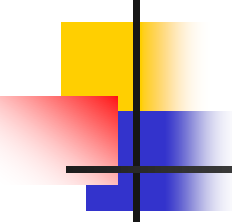
Example 9-10

One such random sample of 200 chips taken recently from the production line contained 14 defective chips. Test at the 5% significance level whether or not the machine needs an adjustment.



Solution 9-10

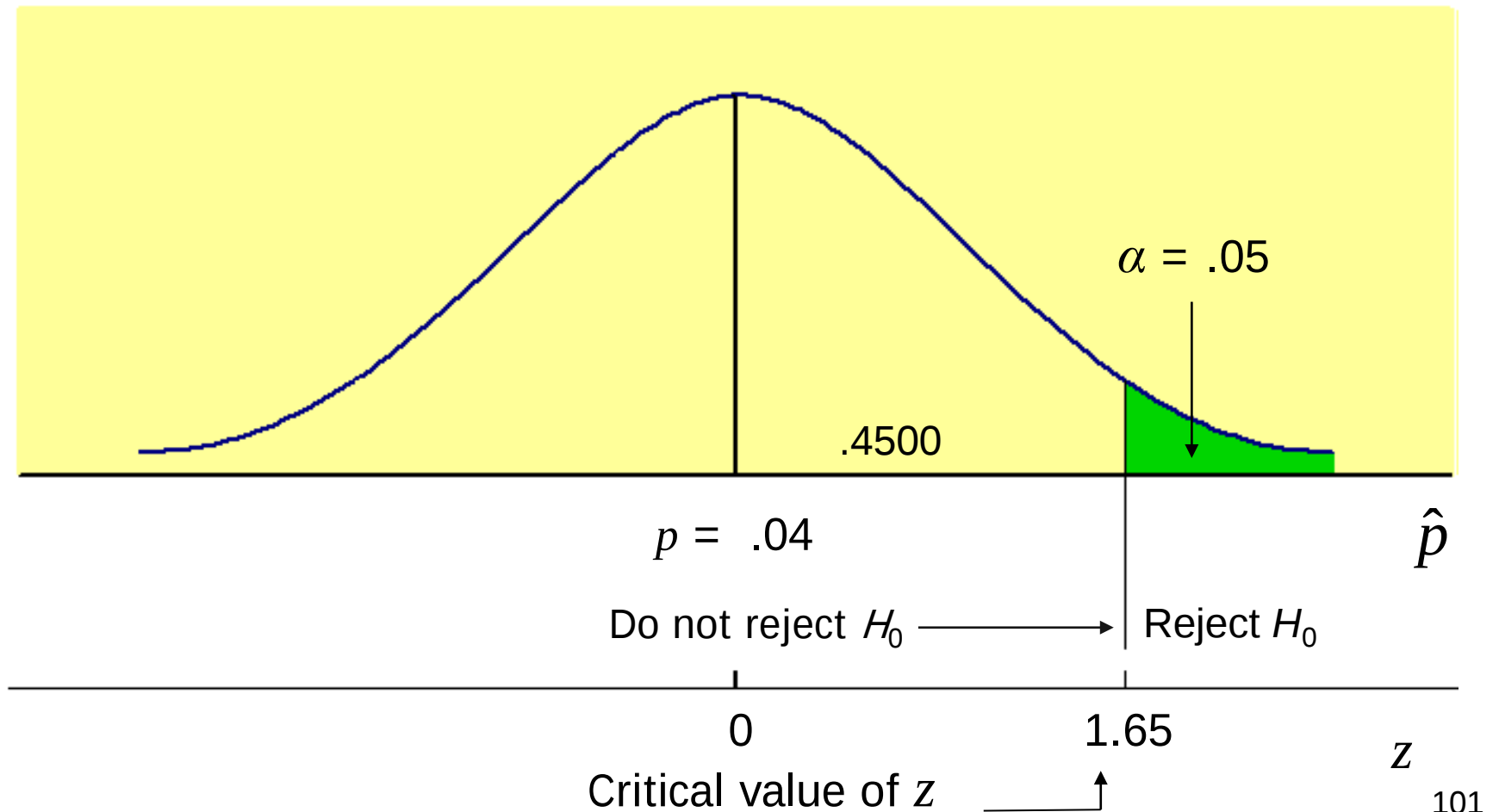
- $H_0: p \leq .04$
 - The machine does not need an adjustment
- $H_1: p > .04$
 - The machine needs an adjustment

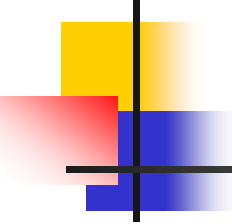


Solution 9-10

- $n = 200$, and $\hat{p} = \frac{14}{200} = .07$
- $np = 200(.04) = 8$
- $nq = 200(.96) = 192$
- $\alpha = .05$
- Area in the right tail = $\alpha = .05$
- The critical value of z is 1.65

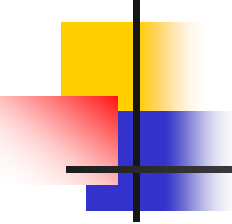
Figure 9.16





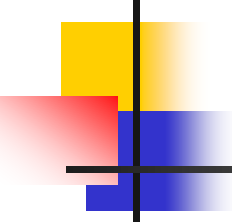
Solution 9-10

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.04)(.96)}{200}} = .01385641 \quad \text{From } H_0$$
$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.07 - .04}{.01385641} = 2.17$$



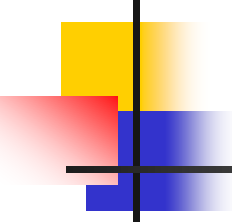
Solution 9-10

- The value of the test statistic $z = 2.17$
 - It is greater than the critical value of z
 - It falls in the rejection region
- Therefore, we reject H_0



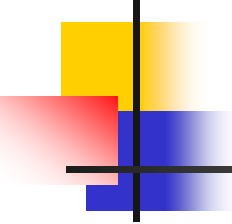
Example 9-11

Direct Mailing Company sells computers and computer parts by mail. The company claims that at least 90% of all orders are mailed within 72 hours after they are received. The quality control department at the company often takes samples to check if this claim is valid. A recently taken sample of 150 orders showed that 129 of them were mailed within 72 hours. Do you think the company's claim is true? Use a 2.5% significance level.



Solution 9-11

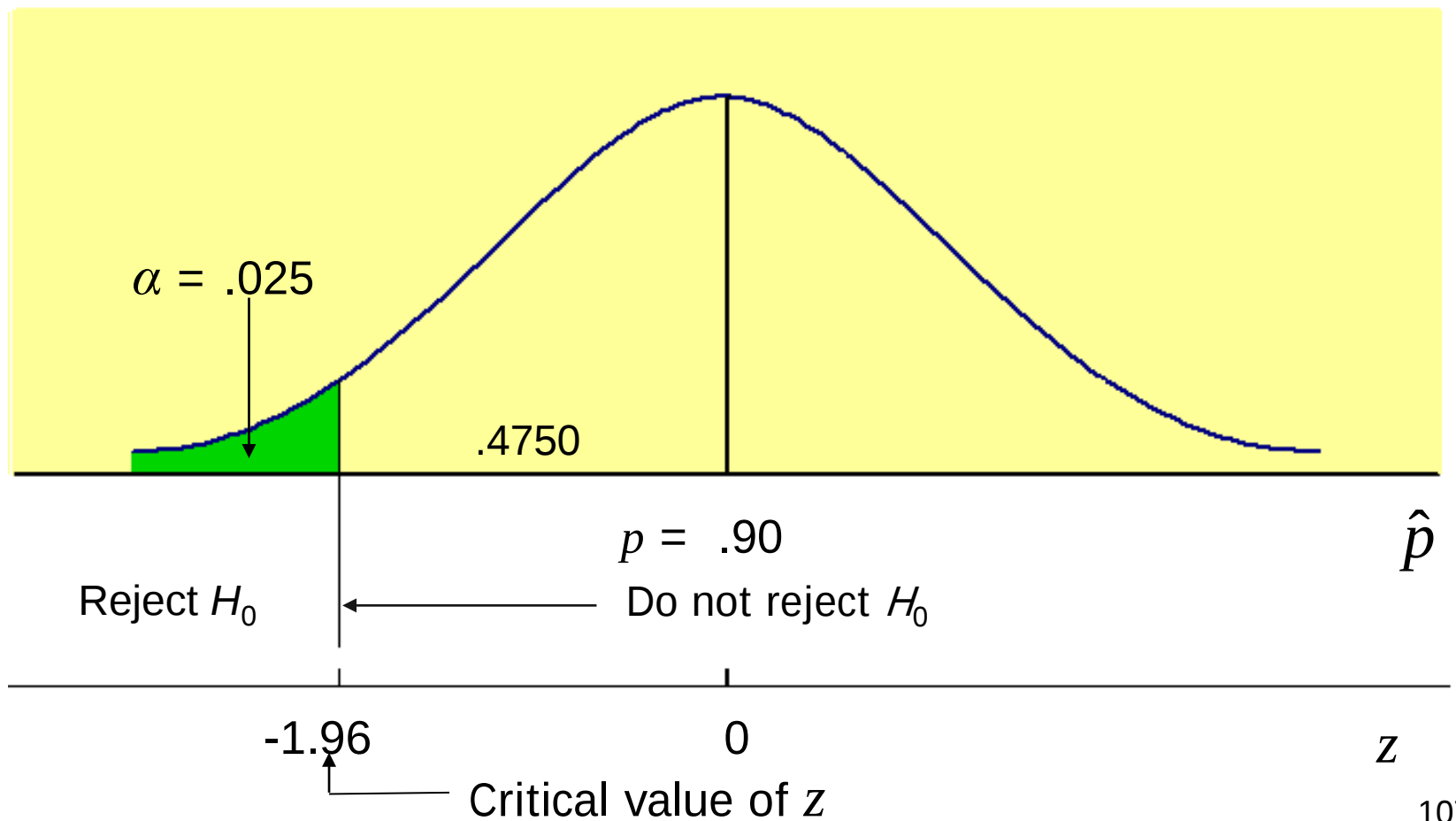
- $H_0: p \geq .90$
 - The company's claim is true
- $H_1: p < .90$
 - The company's claim is false

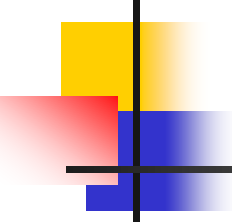


Solution 9-11

- $\alpha = .025.$
- $np = 150(.90) = 135$
- $nq = 150(.10) = 15$
- Both np and nq are greater than 5
- The sample size is large
- Consequently, we use the normal distribution to make the hypothesis test about p
- The critical value of z is -1.96

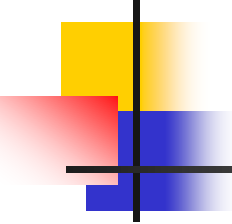
Figure 9.17





Solution 9-11

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.90)(.10)}{150}} = .02449490 \quad \text{From } H_0$$
$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.86 - .90}{.02449490} = -1.63$$



Solution 9-11

- The value of the test statistic $z = -1.63$
 - It is greater than the critical value of z
 - It falls in the nonrejection region
- Therefore, we fail to reject H_0